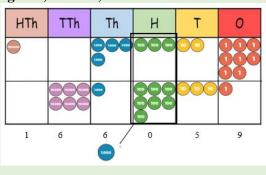


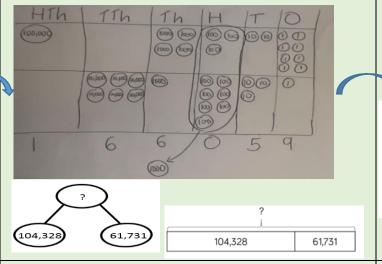
## Add numbers with more than 4 digits

Place value counters are the most effective concrete resources when adding numbers with more than 4 digits, progressing from no exchange to then with exchanging. Ensure pupils write out their caclulations alongside concrete resources so that they can see the link to the written method. The exchange should be shown at the bottom. At this stage, children should be encouraged to work in the abstract, using the **column method** to add larger numbers efficiently.

e.g. 104,328 + 61,731 =



Pupils can draw out their place value counters. Once pupils have secured this skill, they should be able to use part-whole and bar models more efficiently.



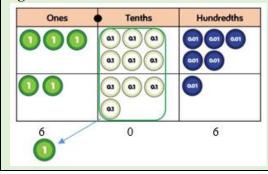
104,328 + 61,731 =



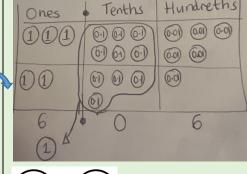
## Add with up to 3 decimal places

\*Encourage pupils to "fill" empty columns with trailing zeros to promote an understanding of using the zero as a placeholder and making it easier to see how the numbers line up. Place value counters and plain counters on a place value grid are the most effective manipulatives when adding decimals with 1, 2 and then 3 decimal places using the **column method**. Ensure children have experience of adding decimals with the same and different number of decimal places\*, progressing from no exchange to then with exchanging. The exchange should be shown at the bottom.

e.g. 3.65 + 2.41 =

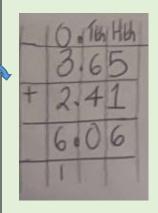


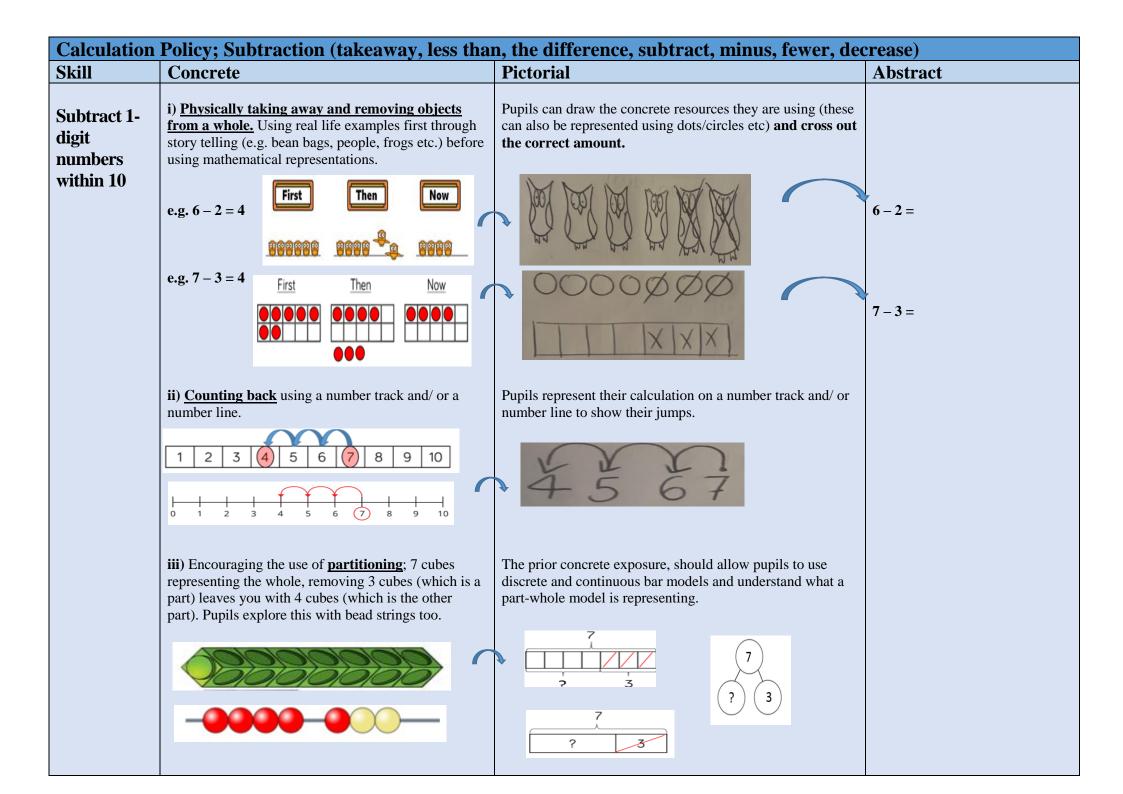
Pupils can draw out their place value counters. Once pupils have secured this skill, they should be able to use part-whole and bar models more efficiently.





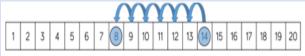
3.65 + 2.41=



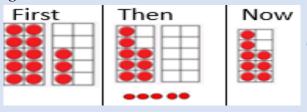


Subtract 1 and 2-digit numbers to 20 (including crossing 10) i) <u>Counting back</u> by using a number track/ number line and physically removing the amount from the whole.

e.g 14 - 6 =



e.g 13 - 5 =



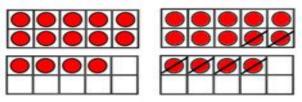
**ii)** Finding the difference (other real life examples can be used such as sticks, pebbles, girls and boys etc)

$$e.g. 12 - 8 =$$

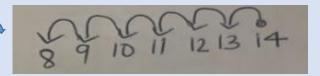


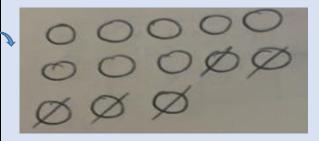
**iii)** Subtract ones by making 10 with the aid of ten frames.

**e.g.** 
$$14 - 6 =$$



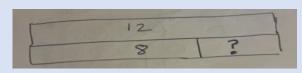
Pupils can draw the concrete resources they are using **and cross out the correct amount.** 



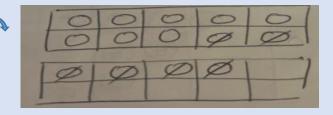


Pupils can draw out the cubes; which in turn should support the use of discrete and continuous bar models.





Pupils to represent the ten frame pictorially and discuss what they did to make 10.





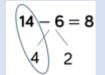
14 - 6 =



12 – 8 =

### 14 - 6 =

Pupils to show how they can make 10 by partitioning the subtrahend.



$$14 - 4 = 10$$
  
 $10 - 2 = 8$ 

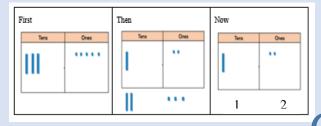
Subtract 1 and 2-digit numbers to 100

\*\*

From Year 3, pupils use the formal written method of column subtraction alongside concrete and pictorial representations so that they can see the link to the written method. The exchange should be shown at the top.

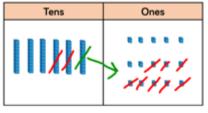
Pupils use dienes and place value counters to subtract 1 and 2-digit numbers to 100 by physically removing the manipulatives; first without exchange and then with exchanging. For subtraction, pupils only need to make the number being subtracted from.

$$e.g. 35 - 23 =$$

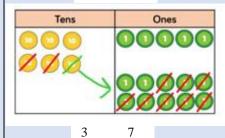




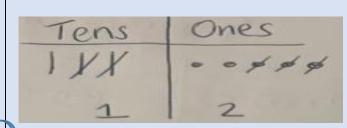
$$e.g. 65 - 28 =$$

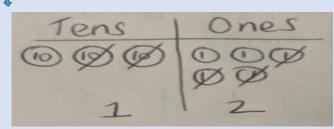


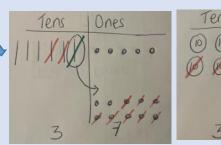
3

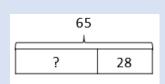


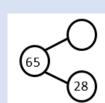
Pupils can represent the dienes using lines and dots and/ or draw out their place value counters. Once pupils are able to understand how to solve the subtraction, they should be able to use part-whole and bar models more efficiently.







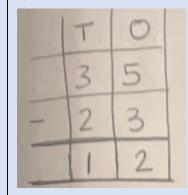




Ones

25		22	
77	_	<i>/.</i> 7	=

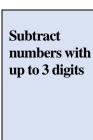
\*\*





\*\*

	T	0
	56	15
-	2	8
	3	7

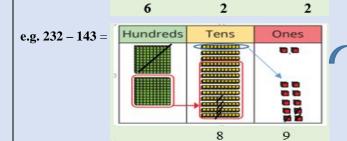


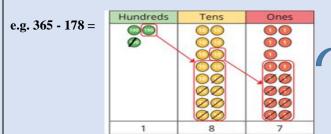
Pupils use dienes and/ or place value counters to support their understanding of the formal written method of column subtraction, first without exchange and then with exchanging (across a 10 and 100, respectively). Ensure pupils write out their caclulations alongside concrete resources so that they can see the link to the written method. The exchange should be shown at the top.

e.g. 769 - 147 =Hundreds Tens Ones 

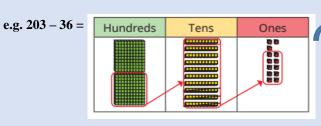
000

 $\bigcirc$ 

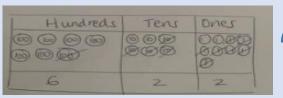




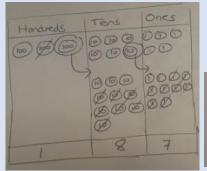
Pupils exchange from the hundreds column to the ones column in a two-part exchange because there are no tens in the original number. Make sure children exchange 1 hundred for 10 tens before exchanging one of those tens for 10 ones.

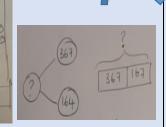


Pupils can represent the dienes using sqaures, lines and dots and/ or draw out their place value counters. Once pupils have secured this skill, they should be able to use part-whole and bar models more efficiently.

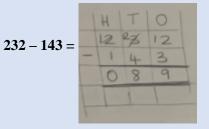


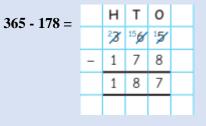






		Н	Т	0	
769 – 147		7	6	9	
105 117	_	1	4	7	
		6	2	2	





			Н	!	O	
			12/	Ď	<sup>1</sup> 3	
,	202 26	-		3	6	
	203 – 36 =		1	6	7	

Hundreds	Tens	ones
	>11111	000
	Darki	700000
1	6	7

Subtract numbers with up to 4 digits	Pupils continue to use dienes and/ or place value counters to support their understanding of the <u>formal written</u> <u>method of column subtraction</u> , first without exchange and then with exchanging (this will include one exchange and more than one exchange, including two-part exchanges). Ensure pupils write out their caclulations alongside concrete resources so that they can see the link to the written method. The exchange should be shown at the top.	Pupils can represent the dienes using cubes, squares, lines and dots and/ or draw out their place value counters. Once pupils have secured this skill, they should be able to use part-whole and bar models more efficiently.	
	e.g. 3,454 – 1,224 = Th H T O O O O O O O O O O O O O O O O O O	Th H T O	3,454 – 1,224 = 3 4 5 4 - 1 2 2 4 2 2 3 0
	e.g. 3,416 – 1,223 = Thousands Hundreds Tens Ones  2 1 9 3	Th H T O	3,416 – 1,223 = 3 3/4 11 6 - 1 2 2 3 2 1 9 3
	e.g. 3,206 – 2,148=  Th		3,206 – 2,148= Th H T O 3 12 9 16 - 2 1 4 8 1 0 5 8
		3,206	

Subtract
numbers with
more than 4
digits

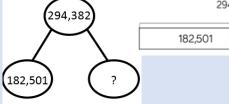
Place value are the most effective concrete resources when subtracting numbers with more than 4 digits, progressing from no exchange to then with exchanging. Ensure pupils write out their caclulations alongside concrete resources so that they can see the link to the written method. The exchange should be shown at the top. At this stage, children should be encouraged to work in the abstract, using the **column method** to subtract larger numbers efficiently.

e.g. 294, 382 - 182, 501 =

HTh	TTh	Th	Н	Т	0
90	<b>O</b> ØØ	900	000	000	00
	000	Ø	000	000	1
1	XUXUXU		ØØØ		
-			0		
1	1	1	8	8	1

Pupils can draw out their place value counters. Once pupils have secured this skill, they should be able to use partwhole and bar models more efficiently.





294, 382 – 182, 501 =

						-	-
<b>*</b>		2	9	3 <b>/</b>	13	8	2
	-	1	8	2	5	0	1
		1	1	1	8	8	1

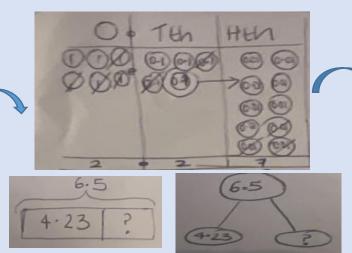
Hth TTh Th H T O

## Subtract with up to 3 decimal places

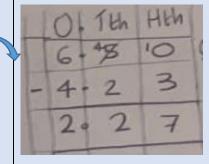
**Show Pupils** how to "fill" empty columns with zeros, which will support them when exchanges are required. They need to be secure with the fact that, for example, 6 and 6.0 have the same numerical value, as do 6.5 and 6.50 and so on.

Place value counters are the most effective manipulatives when subtracting decimals with 1, 2 and then 3 decimal places using the **column method**. Ensure children have experience of subtracting decimals with the same and different number of decimal places\*, progressing from no exchange to then with exchanging. The exchange should be shown at the top.

Pupils can draw out their place value counters. Once pupils have secured this skill, they should be able to use partwhole and bar models more efficiently.



6.5 - 4.23 =



Skill	Concrete	nultiplied by, the product of, groups of, lots of,  Pictorial	Abstract
SKIII	Concrete	1 ictoriai	Abstract
Solve 1- step problems using multiplication ***	Where possible, use real life examples alongside counters, cubes, peg boards etc  e.g. One bag holds 5 apples. How many apples do 4 bags hold?	Pupils can represent the practical resources using dots, crosses or even draw the picture.	One bag holds 5 apples. How many apples do 4 bags hold?
In Year 2, pupils are introduced to the multiplication symbol $5 \times 4 = 20$ Or $4 \times 5 = 20$	i) Repeated addition  There are 4 equal groups.  There are 5 apples in each group.  There are 20 apples altogether. $5+5+5+5=20$	(26) (86) (86) (86)	*** 4 x 5 == 4 x 5 5 + 5 + 5 + 5 =
	**) Number line to show the new set of anoung	Dunile to manage at this mistorially planted to a manhor line	
	ii) Number line to show the repeated groups	Pupils to represent this pictorially alongside a number line.	
	Helix	0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20	
	There are 5 apples in each column. There are 4 rows. There are 20 apples altogether.	Once pupils are able to understand the concept of the repeated addition/ multiplication, they should be able to use	
	Or There are 4 apples in each row. There are 5 rows. There are 20 apples altogether.	other representations (i.e. bar models) more efficiently.	

### Multiply 2digit numbers by 1-digit numbers

### i) Partition to multiply

Pupils use dienes and place value counters to support their understanding, first without exchange and then with exchanging. The 2-digit number is partitioned into tens and ones, both are multiplied by the 1-digit number and then the partial products are added to find the total product.

Pupils can represent the dienes using lines and dots and/ or draw out their place value counters. Once pupils have secured this skill, they should be able to use other representations (e.g. part-whole models) more efficiently.



Tens	Ones
000	1
000	0
000	1
00 00	1

Tens Ones

21 x 4 =

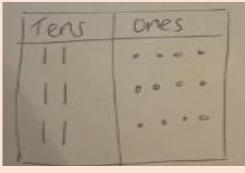
2 tens x 4 = 8 tens 1 one x 4 = 4 ones 80 + 4 = 84 21 x 4 = 84

2 tens x 4 = 8 tens 1 one x 4 = 4 ones 80 + 4 = 84 21 x 4 = 84

e.g. 24 x 3 =

Tens	Ones

2 tens x 3 = 6 tens 4 ones x 3 = 12 ones 60 + 12 = 72 24 x 3 = 72





24 x 3 =

2 tens x 3 = 6 tens 4 ones x 3 = 12 ones 60 + 12 = 72 24 x 3 = 72

### ii) Formal methods

In Year 3, pupils are introduced to the formal methods of multiplication. Place value counters/ dienes are used alongside the formal written method/ abstract calculations. Pupils progress from using the <u>expanded method</u> to the <u>short multiplication method</u>, completing calculations where there are no exchanges to those with exchanges (ensure the exchange is shown along the bottom).

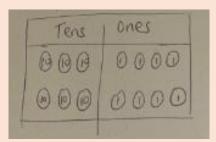
 $e.g 34 \times 2 =$ 

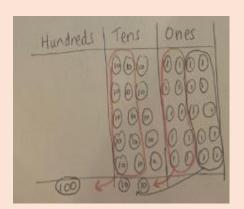
Tens	Ones
000	0000
000	0000

e.g.  $34 \times 5 =$ 

Hundreds	Tens	Ones
	000	0000
	000	0000
	000	0000
	000	0000
	000	0000
0	20	

Pupils can represent the dienes using lines and dots and/ or draw out their place value counters. .



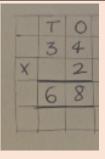




### **Expanded method**

	Т	0	
	3	4	
×		2	
		8	$(2 \times 4 = 8)$
+	6	0	(2 x 30 = 60)
	6	8	

### **Short multiplication method**



34 x 5 = Expanded method

	н	Т	0	
		3	4	
×			5	
		2	0	(5 × 4)
+	1	5	0	(5 × 30)
	1	7	0	

### **Short multiplication method**

	н	Т	0	
		3	4	
×			5	
	1	7	0	
	1	2		

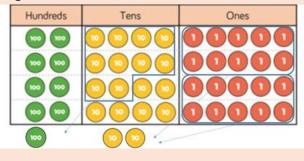
Multiply 3digit numbers by 1-digit numbers

### **Short multiplication**

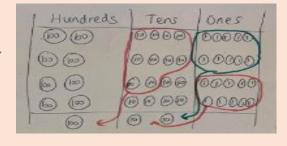
Place value counters/ dienes are used alongside the formal written method/ abstract calculations. Pupils use **short multiplication method**, completing calculations where there are no exchanges to those with exchanges (made for ones, tens and hundreds shown along the bottom).

At this stage, pupils should be encouraged to work in the abstract, using the **short multiplication method**.

e.g. 245 x 4 =



Pupils can represent the dienes using lines and dots and/ or draw out their place value counters.



245 x 4 =

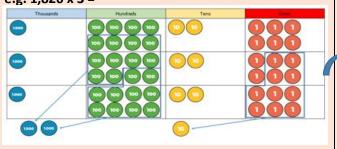
	н	Т	О
	2	4	5
×			4
	9	8	О
		_	_

Multiply 4digit numbers by 1-digit numbers

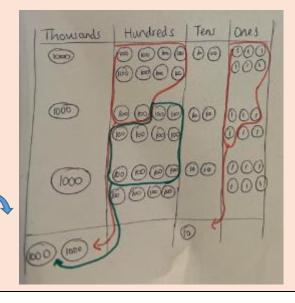
### **Short multiplication**

Place value counters are used alongside the formal written method/ abstract calculations. Pupils use the **short multiplication method**, completing calculations where there are no exchanges to those with exchanges (made for ones, tens, hundreds and thousands shown along the bottom). Pupils should be encouraged to work in the abstract, using the **short** written method.

e.g. 1,826 x 3 =



Pupils can draw out their place value counters.



1.826 x 3 =

Th H T O

1 8 2 6

× 3

5 4 7 8

2 1

### **Multiply 2**digit numbers by 2-digit numbers

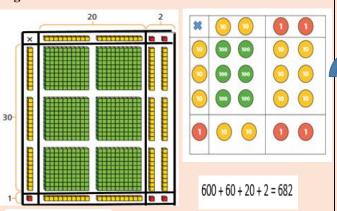
### \*\*\*\*

(Once pupils notice how the subtotals match the totals of parts in the grid method, pupil do not need to show a breakdown of their calculations on the side).

### Formal methods

Place value counters/ dienes are used alongside the formal written method/ abstract calculations. Pupils progress from using the grid method to the long multiplication method, completing calculations with and without exchanges (shown along the bottom). Pupils must understand the importance of the zero as a placeholder.

 $e.g. 22 \times 31 =$ 



There are 6 hundreds

There are 8 tens.

There are 2 ones.

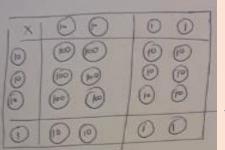
600 + 80 + 2 = 682

### e.g. 27 x 46=

e.g. Z/	N 70-	
х	<b>10 10</b>	000000
10	<b>••••</b>	
100		
<u> </u>		
0	<b>60 60</b>	
•	00 00	0000000
	00 00	0000000
	000	0000000
	10 10	0000000
	000	0000000
1	00 00	0000000

800 + 280 + 120 + 42 = 1,242

Pupils can represent the dienes using lines and dots and/ or draw out their place value counters.



# 1 (D) (D) 00

### **Grid Method**

Х	20	2
30	600	60
1	20	2

600 + 60 + 20 + 2 = 68222 x 31 = 682

### \*\*\*\*\* Long Multiplication Method

	н	Т	0	
		2	2	
×		3	1	
		2	2	(22 x 1)
+	6	6	0	(22 x 30)
	6	8	2	

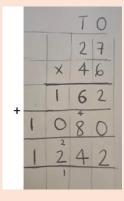
### **Grid Method**

	Х	20	7
	40	800	280
	6	120	42

800 + 280 + 120 + 42 = 1,242

27 x 46 = 1,242

### **Long Multiplication Method**



### Multiply 3digit numbers by 2-digit numbers

### **Long multiplication**

Place value counters are used alongside the formal written method/ abstract calculations. Pupils use the <u>long</u> <u>multiplication method</u> to complete calculations with and without exchanges (shown along the bottom). Pupils must understand the importance of the zero as a placeholder. Pupils must have a secure understanding of the previous methods and may need to refer back to the grid method if required.

e.g.  $234 \times 32 =$ 

100	100	10	0	10	0000
					0000
	200				
					0000
	1000	100 100 100 100 100 100 100 100 100 100	000 000 000 000 000 000 000 000 000 00	100 100 100 100 100 100 100 100 100 100	100 100 100 100 100 100 100 100 100 100

6000 + 900 + 120 + 400 + 60 + 8 = 7,488

Pupils can draw out their place value counters.



### **Grid Method (if required)**

	×	200	30	4
7	30	6,000	900	120
	2	400	60	8

6000 + 900 + 120 + 400 + 60 + 8 = 7,488 234 x 32 = 7,488

### **Long Multiplication Method**

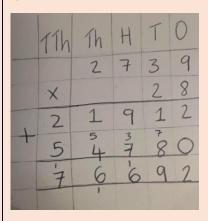


### Multiply 4digit numbers by 2-digit numbers

### **Long multiplication**

At this stage, pupils should be encouraged to work in the abstract, using the <u>long multiplication method</u> to complete calculations with and without exchanges (shown along the bottom). Pupils must understand the importance of the zero as a placeholder. Pupils must have a secure understanding of the previous methods and if required, may need to refer back to using place value counters and/ or the grid method.

2, 739 x 28=



Calculation Policy: Division (share, group, divide, divided by, half, divisor, dividend, remainder)			
Skill	Concrete	Pictorial	Abstract
Solve 1-step problems using division  *****  In Year 2, pupils are introduced to the division symbol	Where possible, use real life examples alongside counters, cubes, bead strings, hoops for sorting groups etc  e) Sharing Pupils solve problems by sharing amounts into equal groups. e.g. There are 20 apples altogether. They are shared equally between 5 bags. How many apples are in each bag?  There are 20 apples. They are shared equally between 5 bags. There are 4 apples in each bag.	Pupils can represent the practical resources using dots, crosses or even draw the picture. Once pupils have secured this skill, they should be able to use other representations (e.g. bar- models) more efficiently, when encountered.	There are 20 apples altogether. They are shared equally between 5 bags. How many apples are in each bag?  ****** 20 ÷ 5 = 4
$20 \div 5 = 4$ $20 \div 4 = 5$	ii) Grouping Pupils solve problems by grouping and counting the number of groups.  e.g. There are 20 apples altogether. They are put in groups of 5. How many groups are there?  There are 20 apples altogether. They are put in groups of 5. There are 4 equal groups of 5 apples.		There are 20 apples altogether. They are put in bags of 5. How many bags are there?  ****** 20 ÷ 4 = 5
	iii) Repeated subtraction Grouping in the above step encourages pupils to count in multiples and links to repeated subtraction on a number line.  -5 -5 -5 -5 -5	01254567891011121314151614181420	

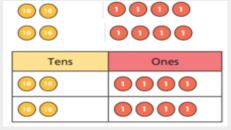
Divide 2digit numbers by 1-digit numbers through sharing

(¹ liaise with Mathematics team/ Team Leader for further clarification) Pupils use dienes or place value counters to support their understanding of partitioning a number into tens and ones and sharing into equal groups. It is important that pupils divide the tens first and then the ones. Pupils should start with the equipment outside the place value grid before sharing the tens and ones equally between the rows. Pupils progress from completing calculations with no exchange, to exchanging and then those with remainders.

(¹ It is important that pupils make sense of the place value counters and write or talk through calculations to show an understanding of the process).

### i) Sharing with no exchange

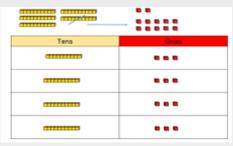
**e.g.**  $48 \div 2 = 24$ 



$$48 \div 2 = 
48 = 40 + 8 
40 \div 2 = 20 
8 \div 2 = 4 
20 + 4 = 24$$

### ii) Sharing with exchange

e.g.  $52 \div 4 = 13$ 



52 ÷ 4=
52 = 40 + 12
$40 \div 4 = 10$
$12 \div 4 = 3$
10 + 3 = 13

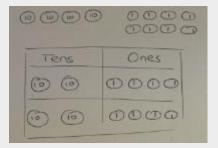
### iii) Sharing with remainders

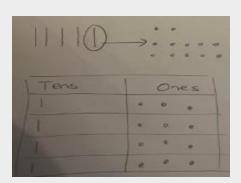
e.g.  $53 \div 4 = 13 \text{ r } 1$ 



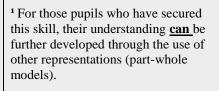
$$53 \div 4 = 
53 = 40 + 13 
40 \div 4 = 10 
12 \div 4 = 3 
1 remainder 
10 + 3 = 13 13 
r1$$

Pupils can represent the dienes using lines and dots and/ or draw out their place value counters.

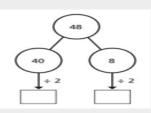




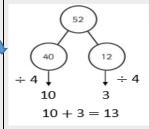




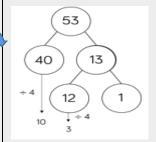




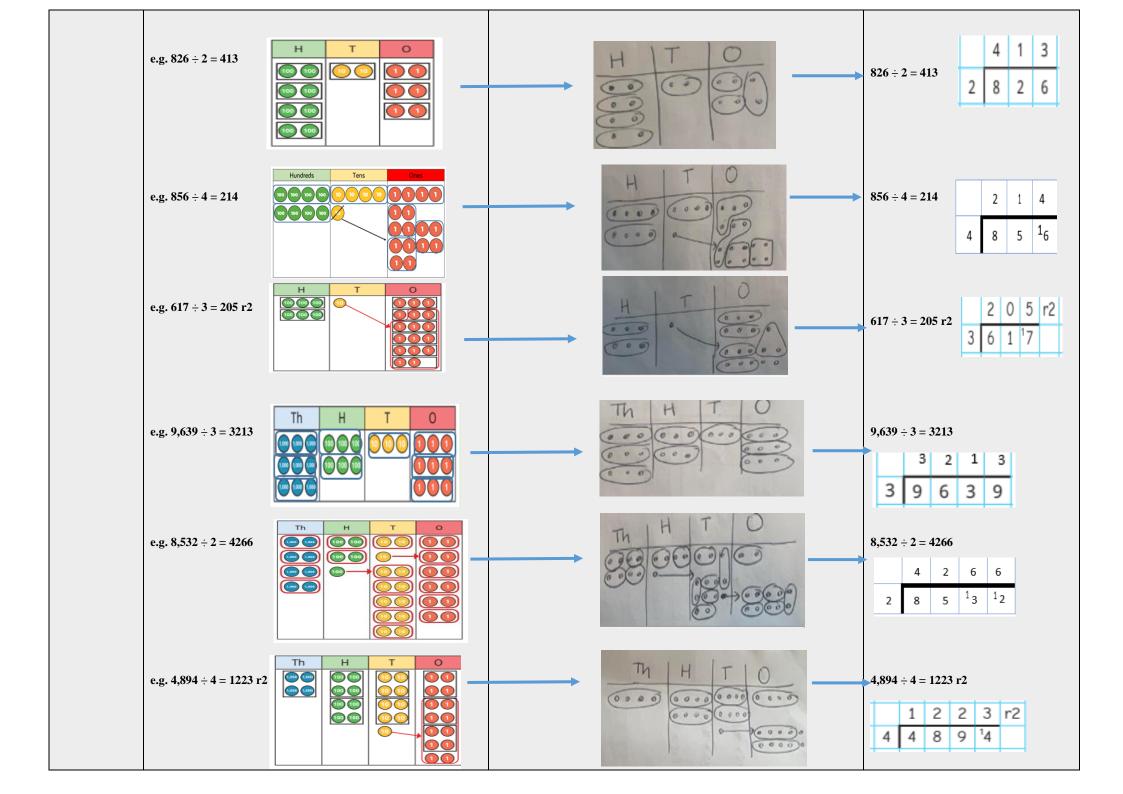




$$53 \div 4 = 13 \text{ r } 1$$



If required, pupils can draw place value Pupils can represent the dienes using lines and dots and/ or draw Divide up to <sup>2</sup> Pupils in Year 3 are introduced to the formal written counters under the dividend to support out their place value counters. <sup>1</sup> For those pupils who have secured 4-digit method of short division. Place value counters/ dienes are them with the transition from the this skill, their understanding can be further developed through the used alongside the formal written method/ abstract concrete/ pictorial to the abstract. numbers by use of other representations (e.g. part-whole models- refer to the calculations. When using the short division method, **pupils use** 1-digit above examples). **grouping**- starting with the largest place value, they group by numbers the divisor (using divisors according to the year group using the **expectation for times table).** Remainders can also be seen as they are left ungrouped. formal First, pupils use short division to divide a 2-digit number by a written 1-digit number, initially without an exchange, then with an method of exchange and those which leave remainders. They then short explore the same set of skills with 3- digit and 4- digit division numbers (depending on the year group expectations for size/ place value of numbers).  $39 \div 3 = 13$ e.g.  $39 \div 3 = 13$ 3 <sup>2</sup> Pupils may need to be 3 3 reminded of the transition between sharing and grouping. If so,  $52 \div 4 = 13$ use the above method (with numbers 3 according to year group expectations). e.g.  $52 \div 4 = 13$ Once pupils have a secure conceptual understanding of this method, pupils should be encouraged to move away  $37 \div 3 = 12 \text{ r } 1$ from the concrete and e.g.  $37 \div 3 = 12 \text{ r } 1$ pictorial when 2 r1 dividing numbers with multiple exchanges.



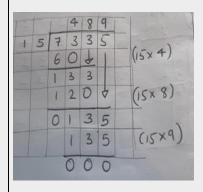
### Divide up to 4digit numbers by 2-digit numbers using the formal written method of <u>long</u> division

<sup>3</sup> Pupils should also be shown how short division can be used to calculate the above, interpreting remainders according to the context, but long division is preferred as we may not be strong with our multiples for all 2 digit numbers. (Liaise with Mathematics team/ Team Leader for further clarification)

<sup>3</sup>Pupils in Year 6 are taught the <u>formal written methods of long division</u> when dividing up to 4- digit numbers by 2-digit numbers. Pupils complete calculations without remainders and then with remainders using the <u>pulling down method</u>. Remainders can be interpreted as whole numbers, fractions or by rounding, as appropriate foe the context. Some pupils may find it helpful to list the multiples of the divisor before attempting the calculation (up to x12).

At this stage, pupils should be encouraged to work in the abstract.

**e.g.**  $7,335 \div 15 = 489$ 



e.g. 
$$372 \div 15 = 24 \text{ r } 12 \text{ (24} \frac{4}{5})$$

