



Pupils continue to use dienes and/ or place value counters to support their understanding of the formal written method of column addition, first without exchange and then with exchanging (this will include across a 10 and 100). Ensure pupils write out their caclulations alongside concrete resources so that they can see the link to the written method. The exchange should be shown at the bottom.

e.g. $208+313=$

e.g. $466+353=$

e.g $367+164=$


Pupils can represent the dienes using sqaures, lines and dots and/or draw out their place value counters. Once pupils have secured this skill, they should be able to use part-whole and bar models more efficiently.



## Add numbers

 with up to 4 digitsPupils continue to use dienes and/ or place value counters to support their understanding of the formal written method of column addition, first without exchange and then with exchanging (this will include one exchange and more than one exchange). Ensure pupils write out their caclulations alongside concrete resources so that they can see the link to the written method. The exchange should be shown at the bottom.
e.g. 3,356 + 2,435

$\mid \mathbf{1 , 3 7 8}+\mathbf{2 , 1 4 8}=$
1378
$+2148$
3526

Pupils can represent the dienes using cubes, sqaures, lines and dots and/or draw out their place value counters. Once pupils have secured this skill, they should be able to use partwhole and bar models more efficiently.
e.g. $\mathbf{3 , 2 5 6}+\mathbf{2 , 5 3 2}=$


$3,356+2,435=$


Th H T 0
e.g. $1,378+2,148=$



## Add numbers

 with more than 4 digits

## 3 decimal

places

## *Encourage

 pupils to "fill" empty columns with trailing zeros to promote an understanding of using the zero as a placeholder and making it easier to see how the numbers line up.Place value counters are the most effective concrete resources when adding numbers with more than 4 digits, progressing from no exchange to then with exchanging. Ensure pupils write out their caclulations alongside concrete resources so that they can see the link to the written method. The exchange should be shown at the bottom. At this stage, children should be encouraged to work in the abstract, using the column method to add larger numbers efficiently.
e.g. $104,328+61,731=$


Place value counters and plain counters on a place value grid are the most effective manipulatives when adding decimals with 1, 2 and then 3 decimal places using the column method. Ensure children have experience of adding decimals with the same and different number of decimal places*, progressing from no exchange to then with exchanging. The exchange should be shown at the bottom.
e.g. $3.65+2.41=$


Pupils can draw out their place value counters. Once pupils have secured this skill, they should be able to use part-whole and bar models more efficiently.


Pupils can draw out their place value counters. Once pupils have secured this skill, they should be able to use part-whole and bar models more efficiently.
$3.65+2.41=$


## Calculation Policy; Subtraction (takeaway, less than, the difference, subtract, minus, fewer, decrease)

## Skill

## Subtract 1-

 digit numbers within 10Concrete
i) Physically taking away and removing objects from a whole. Using real life examples first through story telling (e.g. bean bags, people, frogs etc.) before using mathematical representations.
e.g. $6-2=4$

ii) Counting back using a number track and/ or a number line

iii) Encouraging the use of partitioning; 7 cubes representing the whole, removing 3 cubes (which is a part) leaves you with 4 cubes (which is the other part). Pupils explore this with bead strings too.


Pictorial

Pupils can draw the concrete resources they are using (these can also be represented using dots/circles etc) and cross out the correct amount.


Pupils represent their calculation on a number track and/ or number line to show their jumps.


The prior concrete exposure, should allow pupils to use discrete and continuous bar models and understand what a part-whole model is representing


Subtract 1 and 2-digit numbers to 20 (including crossing 10)
i) Counting back by using a number track/ number line and physically removing the amount from the whole.
e.g $14-6=$
mm

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

e.g $13-5=$

iii) Subtract ones by making 10 with the aid of ten frames.
e.g. $14-6=$



Pupils can draw out the cubes; which in turn should support the use of discrete and continuous bar models.

Pupils to represent the ten frame pictorially and discuss what they did to make 10 .
Pupils can draw the concrete resources they are using and cross out the correct amount.

$14-6=$
Pupils to show how they can make 10 by partitioning the subtrahend.

$14-4=10$
$10-2=8$

## Subtract 1 and

 2-digit numbersto 100
**
From Year 3, pupils use the formal written method of column subtraction alongside concrete and pictorial representations so that they can see the link to the written method.
The exchange
should be shown at the top.

Pupils use dienes and place value counters to subtract 1 and 2-digit numbers to 100 by physically removing the manipulatives; first without exchange and then with exchanging. For subtraction, pupils only need to make the number being subtracted from.
e.g. $35-23=$

e.g. $65-28=$


Pupils can represent the dienes using lines and dots and/ or draw out their place value counters. Once pupils are able to understand how to solve the subtraction, they should be able to use part-whole and bar models more efficiently.

**


## Subtract

 numbers with up to 3 digitsPupils use dienes and/ or place value counters to support their understanding of the formal written method of column subtraction, first without exchange and then with exchanging (across a 10 and 100 , respectively). Ensure pupils write out their caclulations alongside concrete resources so that they can see the link to the written method. The exchange should be shown at the top.
e.g. $769-147=$

e.g. $232-143=$

e.g. $365-178=$


Pupils can represent the dienes using sqaures, lines and dots and/ or draw out their place value counters. Once pupils have secured this skill, they should be able to use part-whole and bar models more efficiently.


365-178 =


$232-143=$


365-178 =
$203-36=$

Pupils exchange from the hundreds column to the ones column in a two-part exchange because there are no tens in the original number. Make sure children exchange 1 hundred for 10 tens before exchanging one of those tens for 10 ones.
e.g. $203-36=$


## Subtract numbers with up to 4 digits

Pupils continue to use dienes and/ or place value counters to support their understanding of the formal written method of column subtraction, first without exchange and then with exchanging (this will include one exchange and more than one exchange, including two-part exchanges). Ensure pupils write out their caclulations alongside concrete resources so that they can see the link to the written method. The exchange should be shown at the top.
e.g. 3,454-1,224 =

$3,416-1,223=$

| Th | $\mathbf{H}$ | $\mathbf{T}$ | $\mathbf{0}$ |
| ---: | ---: | ---: | ---: |
| 3 |  |  |  |
| 1 | 1 | 6 |  |
| -1 | 2 | 2 | 3 |
| 2 | 1 | 9 | 3 |

e.g. $\mathbf{3 , 2 0 6}-\mathbf{2 , 1 4 8}=$


Pupils can represent the dienes using cubes, sqaares, lines and dots and/ or draw out their place value counters. Once pupils have secured this skill, they should be able to use part-whole and bar models more efficiently.
e.g. $\mathbf{3 , 4 1 6}-1,223=$



## Subtract

 numbers with more than 4 digits
## Subtract with

 up to 3 decimal placesShow Pupils how to "fill" empty columns with zeros, which will support them when exchanges are required. They need to be secure with the fact that, for example, 6 and 6.0 have the same numerical value, as do 6.5 and 6.50 and so on.

Place value are the most effective concrete resources when subtracting numbers with more than 4 digits, progressing from no exchange to then with exchanging. Ensure pupils write out their caclulations alongside concrete resources so that they can see the link to the written method. The exchange should be shown at the top. At this stage, children should be encouraged to work in the abstract, using the column method to subtract larger numbers efficiently.
e.g. 294, $382-182,501=$


Place value counters are the most effective manipulatives when subtracting decimals with 1,2 and then 3 decimal places using the column method. Ensure children have experience of subtracting decimals with the same and different number of decimal places*, progressing from no exchange to then with exchanging. The exchange should be shown at the top.

$\qquad$



Pupils can draw out their place value counters. Once pupils have secured this skill, they should be able to use partwhole and bar models more efficiently.

$294,382-182,501=$


Pupils can draw out their place value counters. Once pupils have secured this skill, they should be able to use partwhole and bar models more efficiently.


Calculation Policy: Multiplication (double, times, multiplied by, the product of, groups of, lots of, equal groups)

| Skill | Concrete | Pictorial | Abstract |
| :---: | :---: | :---: | :---: |
| Solve 1- step problems using multiplication $* * *$ <br> In Year 2, pupils are introduced to the multiplication symbol <br> $5 \times 4=20$ <br> Or <br> $4 \times 5=20$ | Where possible, use real life examples alongside counters, cubes, peg boards etc <br> e.g. One bag holds 5 apples. How many apples do 4 bags hold? <br> i) Repeated addition <br> There are 4 equal groups. <br> There are 5 apples in each group. <br> There are 20 apples altogether. $5+5+5+5=20$ <br> ii) Number line to show the repeated groups <br> There are 5 apples in each column. <br> There are 4 rows. <br> There are 20 apples altogether. <br> Or <br> There are 4 apples in each row. <br> There are 5 rows. | Pupils can represent the practical resources using dots, crosses or even draw the picture. <br> Pupils to represent this pictorially alongside a number line. <br> 01234567891011121314151617181920 <br> Once pupils are able to understand the concept of the repeated addition/ multiplication, they should be able to use other representations (i.e. bar models) more efficiently. $\square$ | One bag holds 5 apples. How many apples do 4 bags hold? <br> *** <br> $4 \times 5=$ $5+5+5+5=$ |

Multiply 2digit numbers by 1 -digit numbers

## i) Partition to multiply

Pupils use dienes and place value counters to support their understanding, first without exchange and then with exchanging. The 2 -digit number is partitioned into tens and ones, both are multiplied by the 1-digit number and then the partial products are added to find the total product.

## e.g. $21 \times 4=$



## 2 tens x $3=6$ tens

4 ones $\times 3=12$ ones
$60+12=72$
$24 \times 3=72$

Pupils can represent the dienes using lines and dots and/ or draw out their place value counters. Once pupils have secured this skill, they should be able to use other representations (e.g. part-whole models) more efficiently.


## $21 \times 4=$

2 tens $\times 4=8$ tens
1 one $\times 4=4$ ones
$80+4=84$
$21 \times 4=84$

i) Formal methods

In Year 3, pupils are introduced to the formal methods of multiplication. Place value counters/ dienes are used alongside the formal written method/ abstract calculations. Pupils progress from using the expanded method to the short multiplication method,
completing calculations where there are no exchanges to those with exchanges (ensure the exchange is shown along the bottom).
e.g $34 \times 2=$


## e.g. $34 \times 5=$



Pupils can represent the dienes using lines and dots and/ or draw out their place value counters.

## $34 \times 2=$

## Expanded method



## Short multiplication method



## $34 \times 5=$

Expanded method


## Short multiplication method



## Multiply 3-

 digit numbers by 1 -digit numbersMultiply 4 digit numbers
by 1-digit numbers

## Short multiplication

Place value counters/ dienes are used alongside the formal written method/ abstract calculations. Pupils use short multiplication method, completing calculations where there are no exchanges to those with exchanges (made for ones, tens and hundreds shown along the bottom).
At this stage, pupils should be encouraged to work in the abstract, using the short multiplication method.

## e.g. $245 \times 4=$



Pupils can represent the dienes using lines and dots and/ or draw out their place value counters.

## Short multiplication

Place value counters are used alongside the formal written method/ abstract calculations. Pupils use the short multiplication method, completing calculations where there are no exchanges to those with exchanges (made for ones, tens, hundreds and thousands shown along the bottom). Pupils should be encouraged to work in the abstract, using the short written method.


Pupils can draw out their place value counters.

$245 \times 4=$




## Multiply 2-

 digit numbers by 2 -digit numbers
## *****

(Once pupils notice how the subtotals match the totals of parts in the grid method, pupil do not need to show a breakdown of their calculations on the side).

## Formal methods

Place value counters/ dienes are used alongside the formal written method/ abstract calculations. Pupils progress from using the grid method to the long multiplication method, completing calculations with and without exchanges (shown along the bottom). Pupils must understand the importance of the zero as a placeholder.

$$
\text { e.g. } 22 \times 31=
$$


$600+60+20+2=682$

There are 6 hundred
There are 8 tens.
There are 2 ones.
$600+80+2=682$

## e.g. $27 \times 46=$

|  | OO | (1) (1) |
| :---: | :---: | :---: |
|  | ${ }^{\circ}{ }^{\circ}$ |  |
| - | $\begin{array}{l\|} \hline 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}$ |  |

[^0]Pupils can represent the dienes using lines and dots and/ or draw out their place value counters.

## Grid Method

| $x$ | 20 | 2 |
| :---: | :---: | :---: |
| 30 | 600 | 60 |
| 1 | 20 | 2 |$\quad$| $600+60+20+2=682$ |
| :--- |
| $22 \times 31=682$ |

***** Long Multiplication Method

|  | $H$ | $T$ | $O$ |  |
| :--- | :--- | :--- | :--- | :--- |
|  |  | 2 | 2 |  |
| $\times$ |  | 3 | 1 |  |
|  |  | 2 | 2 |  |
|  | $(22 \times 1)$ |  |  |  |
| $\boldsymbol{+}$ | 6 | 6 | 0 | $(22 \times 30)$ |
|  | 6 | 8 | 2 |  |

Grid Method

| $x$ | 20 | 7 |
| :--- | :--- | :--- |
| 40 | 800 | 280 |
| 6 | 120 | 42 |

$$
800+280+120+42=1,242
$$

$$
27 \times 46=1,242
$$

Long Multiplication Method


Multiply 3digit numbers by 2-digit numbers

Multiply 4digit numbers by 2 -digit numbers

## Long multiplication

Place value counters are used alongside the formal written method/abstract calculations. Pupils use the long multiplication method to complete calculations with and without exchanges (shown along the bottom). Pupils must understand the importance of the zero as a placeholder. Pupils must have a secure understanding of the previous methods and may need to refer back to the grid method if required.

## e.g. $234 \times 32=$


$6000+900+120+400+60+8=7,488$

Long multiplication
At this stage, pupils should be encouraged to work in the abstract, using the long multiplication method to complete calculations with and without exchanges (shown along the bottom). Pupils must understand the importance of the zero as a placeholder. Pupils must have a secure understanding of the previous methods and if required, may need to refer back to using place value counters and/ or the grid method

Pupils can draw out their place value counters.


## Grid Method (if required)

| $\times$ | 200 | 30 | 4 |
| :---: | :---: | :---: | :---: |
| 30 | 6,000 | 900 | 120 |
| 2 | 400 | 60 | 8 |

$6000+900+120+400+60+8=7,488$
$234 \times 32=7,488$

Long Multiplication Method

$2,739 \times 28=$


Calculation Policy: Division (share, group, divide, divided by, half, divisor, dividend, remainder)

\section*{| Skill | Concrete | Pictorial |
| :--- | :--- | :--- |
| Solve 1-step | Where possible, use real life examples alongside counters, cubes, | Pupils can represent the practical resources using dots, crosses or |
|  |  |  |} bead strings, hoops for sorting groups etc

e) Sharing

Pupils solve problems by sharing amounts into equal groups. e.g. There are 20 apples altogether. They are shared equally between 5 bags. How many apples are in each bag?


There are 20 apples.
They are shared equally between 5 bags.
There are 4 apples in each bag.

## ii) Grouping

Pupils solve problems by grouping and counting the number of groups.
e.g. There are 20 apples altogether. They are put in groups of 5 . How many groups are there?


There are 20 apples altogether.
They are put in groups of 5 .
There are 4 equal groups of 5 apples.
iii) Repeated subtraction

Grouping in the above step encourages pupils to count in multiples and links to repeated subtraction on a number line.


Pupils can represent the practical resources using dots, crosses
even draw the picture. Once pupils have secured this skill, they should be able to use other representations (e.g. bar- models) more efficiently, when encountered.
problems using division
******
In Year 2, pupils are introduced
to the division symbol
$20 \div 5=4$
$20 \div 4=5$


## Abstract

There are 20 apples altogether. They are shared equally between 5 bags. How many apples are in each bag?
******
$20 \div 5=4$

There are 20 apples altogether. They are put in bags of 5 . How many bags are there?
******
$20 \div 4=5$

Divide 2-
digit numbers by
1-digit numbers
through
sharing
( ${ }^{1}$ liaise with
Mathematics
team/ Team
Leader for
further
clarification)

Pupils use dienes or place value counters to support their understanding of partitioning a number into tens and ones and sharing into equal groups. It is important that pupils divide the tens first and then the ones. Pupils should start with the equipment outside the place value grid before sharing the tens and ones equally between the rows. Pupils progress from completing calculations with no exchange, to exchanging and then those with remainders.
( ${ }^{1}$ It is important that pupils make sense of the place value counters and write or talk through calculations to show an understanding of the process).
i) Sharing with no exchange
e.g. $48 \div 2=24$

ii) Sharing with exchange
e.g. $52 \div 4=13$


$$
\begin{aligned}
& 52 \div 4= \\
& 52=40+12 \\
& 40 \div 4=10 \\
& 12 \div 4=3 \\
& 10+3=13
\end{aligned}
$$

iii) Sharing with remainders
e.g. $53 \div 4=13$ r 1


$$
\begin{aligned}
& x 3=40+13 \\
& 40 \div 4=10 \\
& 12 \div 4=3
\end{aligned}
$$

1 remainder
$10+3=1313$
r1

Pupils can represent the dienes using lines and dots and/ or draw out their place value counters.
${ }^{1}$ For those pupils who have secured this skill, their understanding can be further developed through the use of other representations (part-whole models).



e.g. $826 \div 2=413$

| H | T | O |  |
| :---: | :---: | :---: | :---: |
| 100 | 100 |  |  |
| 10 |  | 10 | 1 |
| 100 |  | 100 |  |
| 100 |  | 100 | 1 |
| 100 |  | 100 |  |
|  |  | 1 | 1 |

e.g. $856 \div 4=214$
e.g. $617 \div 3=205$ r2

e.g. $9,639 \div 3=3213$

e.g. $8,532 \div 2=4266$

$\qquad$


$9,639 \div 3=3213$

$4,894 \div 4=1223 \mathbf{r 2}$

|  | 1 | 2 | 2 | 3 | $r 2$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 4 | 4 | 8 | 9 | 1 |  |

## Divide up to 4digit numbers

by 2 -digit
numbers using
the formal
written
method of long
division
${ }^{3}$ Pupils should
also be shown
how short
division can be
used to calculate
the above,
interpreting
remainders
according to the
context, but long division is preferred as we may not be strong with our multiples for all 2 digit numbers.
(Liaise with
Mathematics
team/ Team
Leader for further
clarification)
${ }^{3}$ Pupils in Year 6 are taught the formal written methods of long division when dividing up to 4 - digit numbers by 2 -
digit numbers. Pupils complete calculations without
remainders and then with remainders using the pulling down method. Remainders can be interpreted as whole numbers, fractions or by rounding, as appropriate foe the context. Some pupils may find it helpful to list the multiples of the divisor before attempting the calculation (up to x12).

At this stage, pupils should be encouraged to work in the abstract.
e.g. $\mathbf{7 , 3 3 5} \div \mathbf{1 5}=489$

e.g. $372 \div 15=24$ r $12\left(24 \frac{4}{5}\right)$



[^0]:    $800+280+120+42=1,242$

